Optimal level of Government Controls for Electric Vehicle transition

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Abstract

Governments face increasing pressure to meet net zero targets, requiring the electrification of transport and heating. This paper investigates whether a cost-effective, optimal strategy exists for a country and its citizens to transition from internal combustion engine (ICE) vehicles to electric vehicles (EVs). We present a case study of the UK, and comment on how the transition to EVs is happening around the EU.

We propose a novel approach to the policy problem, including elements of mathematical finance and economics. Our model of the demand for new EVs is a function depending on stochastic factors such as fuel costs as well as government controlled policy decisions. The government is assumed to control tax incentives for EV owners and purchase subsidies for new vehicles and they both influence the demand. By computing the total future costs of these strategies and weighing them against potential penalties for failing to meet targets, we are able to write a Hamilton Jacobi Bellman equation which can be solved to identify the best policy pathway.

Our initial results indicate that a significant number of cars might need to be retired early to comply with net-zero targets, unless a fast transition to EV sales is taken over the next ten years. The model provides a structured framework for policymakers to design cost-efficient EV adoption strategies under different economic conditions.

1 Introduction

Governments face constrained budgets while committed to legally binding net-zero targets, so would like to know the best way to do this. We wish to setup a stochastic optimal control problem to find the optimal way for a country and its citizens to transition from internal combustion engine (ICE) vehicles to electric vehicles (EV), and to find the optimal balance between the incentive costs spent by the government versus the potential penalty costs for missing targets. To do this, we will need to derive a dynamic model for how the number of ICE vehicles operating on the roads will change over time, taking into account the sale of new ICE vehicles are purchased and how many old ICE vehicles are retired from service. To calculate the number of sales of ICE cars, we will need to consider the consumer choice to purchase a new PV or a new EV car, and how the incentives and/or costs at that time influence it, in terms of the price paid to purchase, operating/maintenance costs, taxes, and the availability of infrastructure. When deciding to retire old cars, this will primarily depend on the age of the vehicle in question, so from a population perspective, the average age will be used as a proxy to determine how many cars are being retired. We setup a dynamic model to capture the net economic costs to the government depending on the incentives they put in place, and the potential future penalties if they don't meet their obligations.

A key component of our model is capturing the consumer's decision to switch, and there are many studies looking at this problem from different angles. For instance, Moon and Lee (2019) calculates probability of switching by looking at an option to purchase using a decision tree analysis, using the probability of the price going up or down. He et al. (2017) again looks at the decision to switch from ICE to EV, this time using Geometric Brownian motion and dynamic programming to solve the problem. More recently, Falbo et al. (2021) uses a stochastic variable to capture the difference in operating cost between ICE and EV, primarily driven by the difference in fuel cost. This is something we wish to use, although our model will try to capture the full cost

When it comes to looking at the problem faced by the government, in choosing an incentive to get their citizens to transition to a new technology, there have been some similar studies to ours. One example using stochastic optimal control is Yamashita et al. (2013), who model a government whose objective to maximise the production of EV cars, using incentives on purchase price or investment in infrastructure. This type of model does a good job at predicting the trends over the short term given a particular strategy, but it does not have a final goal or terminal time at which the objective must be realised. Our model uses the net zero target date as part of the objective, changing the way the problem is solved. Looking now at the social cost of transitioning, Riesz et al. (2016) build a model to predict the social cost of adopting EVs, and find that a rapid transition might be more cost effective in the long term. One of the few papers tackling the full problem similar to ours is Alogdianakis and Dimitriou (2023), who solve a vast model which tracks not only the number of each type of vehicles, but also environmental factors, production costs, energy costs, resources and government interventions. Given the number of variables included in their model, it is difficult to see how any one of those will influence the final result, making it somewhat of a black box. We believe our approach can provide insight as to what variables can have the most effect.

Here we first derive the equations that describe how our state variables change over time, as well as the objective function that must be minimised in the optimal strategy. We then show how these state variables predict future trends, and what will happen if no incentives are put in place. We will then need to solve the full problem numerically, and aim to demonstrate what effect stochastic uncertainty has on the optimal policy. Eventually, we will fully calibrate the model to UK data before running case studies on European countries.

2 Model

The aim of the government is to transition all cars to EVs, so first we discuss how to predict the number of cars of each type registered as in use on the road. Let us assume that there are $n(t)$ total vehicles registered in use in the country at time t, which in the UK is estimated to be around 33.5 million as of 2023 (see Table 1). First we note that in Table 1 for an established economy such as the UK, the number of cars per capita has been relatively constant over the past ten years. So using the latest population predictions to get a projected UK population $P(t)$, and taking this as an exogenous input to our model, we can estimate the required number of cars in the future as

$$n(t) = \Gamma P(t)$$

where $\Gamma = 0.49$ is the number of cars per capita, which is assumed to be a constant estimated using the historical average. This gives us the total number of cars operating at all future times, but we are also interested in what is the demand for new cars. If new cars are sold at a rate of ν_s per annum, and at the other end cars are retired from use at the rate ν_r per annum, then we can relate this to our projection of n via the equation

$$n'(t) = \nu_s(t) - \nu_r(t). \tag{1}$$

So if the number of retirements is known, we can predict the demand for new cars in the economy. Looking at the data for the UK in Table 1, we see that in recent years the number of cars being retired from use has gone down from around 2 million between 2015 to 2019, to around 1.5 million between 2020 to 2023. This has the knock-on effect that sales of new cars has gone down from around 2.5 million down to 1.7 million. So the general trend is that although the total number of cars on the road has been increasing inline with population increases, with cars being kept road worthy for longer and retirement delayed, the demand for new cars has gone down. We can see this has had an effect on the average age of a car, which has gone from around 8 years old between 2015 and 2019 up to 9.3 as of the latest 2023 data.

The exact number of cars retiring at any time will depend on the distribution of car ages in the population, with those that are older more likely to be retired from use. We define $T_r(t)$ as the expected age of all cars being retired at time t, and our model for this is given by

$$T_r(t) = \frac{T_\infty}{1 + e^{-\gamma_r(t-t_0)}} \tag{2}$$

where $t_0 = \frac{1}{\gamma_r} \log\left(\frac{T_r(0)}{T_\infty} - 1\right)$. We choose this so that over time it tends towards a $T_\infty > T_r(0)$ at some rate γ_r, both T_∞ and γ_r are constants chosen arbitrarily with values that seem reasonable. Now modelling the full distribution of car ages is not feasible, so instead we adopt a simple linear approximation and assume only the cars reaching retirement age are retired, then we may write

$$n(t) = \int_{\tau=0}^{T_r(t)} \Psi(t,\tau) d\tau = \int_{\tau=0}^{T_r(t)} \nu_s(t) + \alpha_n \tau d\tau \tag{3}$$

as the relationship between the total number of cars n and the distribution $\Psi(t,\tau)$ of all different ages τ at time t. If we assume that the distribution stays consistent with age i.e. $\Psi(t+dt, \tau+dt) = \Psi(t,\tau)$, we must have $\nu_s' = -\alpha_n$ to maintain $\frac{\partial \Psi}{\partial \tau} = \alpha_n$ over the entire interval $\tau \in [0, T_r(t)]$. Then we can simply differentiate (3) to give an expression for α_n as

$$\alpha_n = -\frac{n' - T_r' \frac{n}{T_r}}{T_r - \frac{1}{2} T_r'} \tag{4}$$

and the relationship between the number of sales $\nu_s(t)$ and the number of retirements at time t is

$$\nu_r(t) = (\nu_s + \alpha_n T_r)(1 - T_r').$$

Here we assume that the rate at which retirement age increases cannot be faster than one, or $T_r' < 1$. Under these model assumptions, we are also able to easily calculate the average age of cars given the value of other variables

$$a_n(t) = \frac{T_r^2}{n}\left(\frac{1}{2}\nu_s + \frac{1}{3}\alpha_n T_r\right) \tag{5}$$

and by directly differentiating (5) to find

$$a_n'(t) = 1 - \frac{T_r \nu_r}{n} - (\nu_s - \nu_r)\frac{a_n}{n} \tag{6}$$

where the first term comes from the fact that all cars age at the rate 1, the second captures the reduction when ν_r cars retire with average age T_r, and the final term adjusts according to changes in n. The rate at which new cars enter does not affect the average directly as their contribution to the average is zero.

Now let $m(t)$ define the number of ICE vehicles, and $p(t)$ define the number of EVs operating at time t. The ODEs for these two variables are given by

$$m'(t) = \mu_s - \mu_r \tag{7}$$

$$p'(t) = \phi_s - \phi_r \tag{8}$$

and

$$a_m'(t) = 1 - \frac{T_r \mu_r}{m} - (\mu_s - \mu_r)\frac{a_m}{m} \tag{9}$$

$$a_p'(t) = 1 - \frac{T_r \phi_r}{p} - (\phi_s - \phi_r)\frac{a_p}{p} \tag{10}$$

where μ_s, ϕ_s and μ_r, ϕ_r are the number of sales/retirements in ICE/EV respectively, and a_m, a_p are the average ages of ICE/EVs respectively. By tracking the average age of each car type, we are able to estimate the expected number of retirements of each type of car by

$$\hat{\mu}_r = \max\left\{\frac{6a_m m}{T_r^2} - \frac{2m}{T_r}, 0\right\} \tag{11}$$

$$\hat{\phi}_r = \max\left\{\frac{6a_p p}{T_r^2} - \frac{2p}{T_r}, 0\right\}. \tag{12}$$

Given the model assumptions, this number may not be very accurate and even negative in some cases, so in order to keep a consistent model we use the expected number of retirement in total and assign a proportion to each class of car given the size of $\hat{\mu}_r$ and $\hat{\phi}_r$. This is given by

$$\mu_r = \nu_r \frac{\hat{\mu}_r}{\hat{\phi}_r + \hat{\mu}_r} \tag{13}$$

$$\phi_r = \nu_r \frac{\hat{\phi}_r}{\hat{\phi}_r + \hat{\mu}_r}. \tag{14}$$

In practice, we find it better to track a_m and a_n and use the definition of an average to determine a_p:

$$a_p(t) = \frac{1}{p}(a_n n - a_m m). \tag{15}$$

Now consider that the government wishes to target the $\nu_s dt$ owners looking to buy a new car, and can use a subsidy to convince them to choose an EV car as their next purchase. The consumer will make a decision based both on capital cost of the purchase as well as some utility they receive or desire from the item. The utility of this purchase will be influenced by several different factors. Firstly, and one of the most important will be the average cost per unit of distance travelled, and this will depend on the current oil prices in the case of ICE vehicles, or electricity prices for an EV. We note that Falbo et al. (2021) adopted a similar approach in their model of an individual's decision to switch, and they use a stochastic variable to capture the current price difference as the most important influence on the decision. We believe that the way cars are promoted to consumers offers a simplistic cost benefit analysis in view of current prices, and that the average customer in a country is not studying historical prices and neither are they projecting future prices. Secondly, another combined factor will be distance the vehicle can travel before needed to be refilled with fuel, versus the availability of refueling infrastructure, i.e. the distance to petroleum stations or electric charging stations. We expect that these factor will change over time in response to market conditions, as more people buy EV, the petroleum stations serving a minority of users will become uneconomical and close down, increasing the distance to nearest station for the remaining users. Consumers will also likely be aware of the repair or maintenance costs involved in the purchase and the depreciation in value of the two different types of car. Finally, government taxes or road use charges targeted at one type of car may also influence the decision. We then introduce a demand function, λ_s which indicates the demand to choose EV over ICE as a new purchase, and we note here that demand for EV cars is limited by the total number of new cars demanded in the market. We assume that the **demand** function for EV cars depends on:

- $x(t)$ is an uncontrollable variable that captures the user desire for EVs, depending on the total cost of ownership for each vehicle (TCO). This can be calculated as the price difference in TCO between EV and ICE cars without any government intervention at time t. We wish to capture this as a single variable, hence we aggregate this number over potential vehicle classes;
- $c(t)$ is a control that sets incentives such as reduced road charges (i.e. exemption from tolls) and a reduction in road tax or fuel tax. This expresses the reigning control at time t affecting all current EV drivers;
- $\pi(t)$ a control that capture government backed discounts on the capital cost of purchase. These one off payments at a given time point t will depend on the vehicle class and like $x(t)$, they will be taken on an aggregated level;

such that the decision for an average customer when it comes to purchasing a new car is based on a number they calculate as

$$T.EV(t) = x(t) - c(t) - \pi(t).$$

$T.EV(t)$ hence denotes the total incentive for an EV car to be bought at time t.

As is standard in the literature, we make sure the **demand** function satisfies the following properties:

- as costs get very large and positive, $\lambda \to 0$
- as costs get very large and negative, $\lambda \to 1$,
- and λ is monotonically increasing as costs decrease.

hence we propose a simple **demand** function of the form

$$\lambda(x, c, \pi) = \hat{\lambda} \min\left(e^{\beta(x-c-\pi)}, 1\right)$$

where β is the price elasticity of an average citizen in the country. Here the current desire/value of the EV is positive $x > 0$ given the very large capital cost required, so we assume that overall lambda would not be large without the support of positive $c > 0$ and $\pi > 0$ incentives. Typically, x is predicted to decrease over time as the cost of batteries and economies of scale kick in to make EV cars more affordable. It will also be the case that as the transition towards EV takes place, less new FF cars will enter the market, so the average age of the remaining stock will increase. Then given a demand function of this form, we have

$$m'(t) = (1 - \lambda(c, \pi))\nu_s - \mu_r \tag{16}$$

$$p'(t) = \lambda(c, \pi)\nu_s - \phi_r \tag{17}$$

as the change in the number of ICE cars and EV cars operating.

We model the user desire to choose EV over ICE (absent of incentives) using a stochastic differential equation

$$dx(t) = \kappa(\theta - x(t))dt + \sigma dW(t)$$

where σ capture the future uncertainty around fuel cost and infrastructure. We will normally set $\theta < 0$ to capture the long term trend that EV cars will be more desireable than ICE cars, and initially we set $x(0) > 0$ to indicate that the majority of users still choose ICE over EV (at least in the UK).

The government is primarily concerned with the number of ICE vehicles, so we can now express this in terms of an optimal control problem, given the state variables m, a_m and the controls c and π. The cost to the government at time t of implementing the given incentives are

$$\lambda\nu_s\pi dt + (n - m)cdt \tag{18}$$

which is the total discount offered to new sales of EV cars plus the tax incentives offered to all p owners of EV cars. Then they must balance what they spend now on incentives in a bid to avoid penalties at some future date

$$K \cdot m(T)$$

which is just some linear cost K multiplied by the remaining number of ICE cars at time $t = T$. Given the legally binding nature of the net zero agreement, we could view that the government will be required to either ban all remaining fossil fuel cars at which point this will be accompanied by a huge drop in GDP if citizens in the state are no longer able to travel to work etc. Either that or they will need to pay to have all of those ICE vehicles retired from service, which will mean they will likely have to pay above market rates to take them, which in turn will drive demand for EV cars further up the chain, increasing the total social cost.

We assume there is some risk adjusted discount r, then the objective of the government is to minimise the net present value of the expected cost

$$J = \mathbb{E}\left[\int_0^T (\lambda\nu_s\pi dt + (n - m)c)e^{-rt}dt + Km(T)e^{-rT}\right],$$

or to find the optimal policy c, π such that

$$J^* = \min_{c,\pi} \mathbb{E}\left[\int_0^T (\lambda(c, \pi)\nu_s\pi dt + (n - m(t))c)e^{-rt}dt + Km(T)e^{-rT}\right].$$

Here the state variables m is determined by (16) and the value of μ_r is determined using (13). If we now define the value function V as the discounted future income under the optimal policy

$$V(m, a_m, x, t) = \min_{c,\pi} \left[\int_t^T (\lambda m \pi + (n(t) - m)c) e^{-r(s-t)} ds + P m e^{-r(T-t)} \right],$$

then using standard arguments can show V satisfies the HJB equation

$$\begin{aligned} \frac{\partial V}{\partial t} - rV + \min_{c,\pi} \Bigg[& \kappa(\theta - x)\frac{\partial V}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} + (\mu_s - \mu_r)\frac{\partial V}{\partial m} \\ & + \left(1 - \frac{T_r \mu_r}{m} - (\mu_s - \mu_r)\frac{a_m}{m}\right)\frac{\partial V}{\partial a_m} + \lambda \nu_s \pi + (n - m)c \Bigg] = 0. \end{aligned} \tag{19}$$

3 Initial Results

Whilst the full numerical solution is still on going, we present some initial results calibrated to the UK car market, solving the ODEs for m (16) and a_m (9), using the calculated values of ν_s (4) and μ_r (13) for a given $\lambda(t)$ as a function of time. Either λ is chosen to switch over to EV sales quickly, reaching 100% before 2035, or more slowly only reaching 100% by 2050. The main data we use to calibrate the model is in Table 1.

Firstly we show the exogenous inputs to the model in Figure 1. Each of the plots will present values for the past 10 years using historical observations, along with the projected values over the next 40 years. The UK population uses the 2022 ONS historical/predicted populations in years 2020, 2022, 2032, and 2052 to fit a smooth quadratic function through the data. This predicts that UK population will peak around 2060 before starting to reduce. This means that in terms of the demand for cars, the number of cars being retired (top right orange line) will gradually catch up with new sales (top right blue line) over the next 35 years, before exceeding them in 2060. We set the initial average age of retirement in the model $T_r(0) = 18.5$, since this sets the average age in the population at 9.25 (the last recorded average age was 9.3 in 2023). The fact that the average age of cars in the population is going up explains why the number of retirements between 2020 and 2025 is much lower than expected given the number of cars registered. We use a prediction of retirement age that it won't increase by very much over the next few years, but this could have an effect on result once a full analysis is carried out.

In Figure 2 we show our first prediction of the number of ICE cars and EV cars registered on the road, using a fast transition to EV sales over the next 5 to 10 years (see bottom right for the percentage sales of EV in each year). In this case we see that the UK would be very close to reaching the net-zero target of zero ICE cars by 2050, with less than a million cars still operating (orange line top left) and the average age of those cars would be between 20 to 25 years (see bottom right of the figure), making them cheap to remove through scrappage schemes. We see that the majority of the retirements are taken up by ICE cars until around 2040 when the current EV cars being sold between 2020 to 2025 start to reach retirement age. This model predicts there will still be a few hundred thousand cars running until 2060, which does not seem unreasonable, but it is likely those cars will be retired earlier due to environmental factors (lack of filling stations etc).

Finally in Figure 3 we present the same set of results as Figure 2, but this time using a slower transition of EV sales over the next 25 years (see bottom right for the percentage sales of EV in each year). Now we see that the UK would be some way off reaching net-zero by 2050, with more than five million cars still operating (orange line top left) and the average age of those cars would now be much lower at 15 years (see bottom right of the figure). This would be expected since ICE cars are still being sold in significant number between 2035-2040, meaning that there will be cars operating with an age less than 15 years old. If this was the situation in 2050, replacing those ICE cars with EV would not be easy given that the population driving those cars would not be likely to be able to afford new EV cars. The number of cars operating m at 2050, and the average age of those cars a_m, seems to be reasonable given the policy adopted in sales. If the

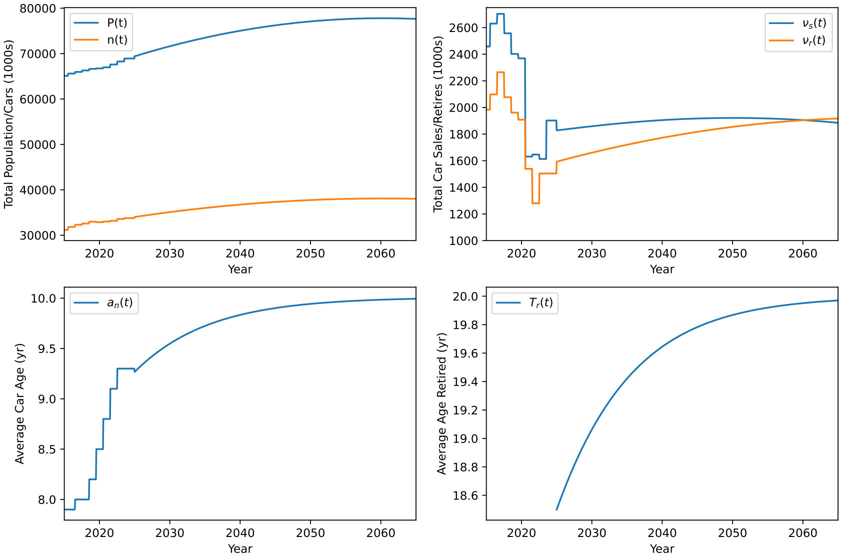

Figure 1: UK model historical data and calculated model inputs. Top left we have UK population projections and expected number of cars required to satisfy demand, using simple linear relationship $n(t) = \Gamma P(t)$. Top right is the predicted demand for sales/retires given average age of car population. Bottom left is the projected average car age of all cars, and bottom right is the our projected retirement age for cars.

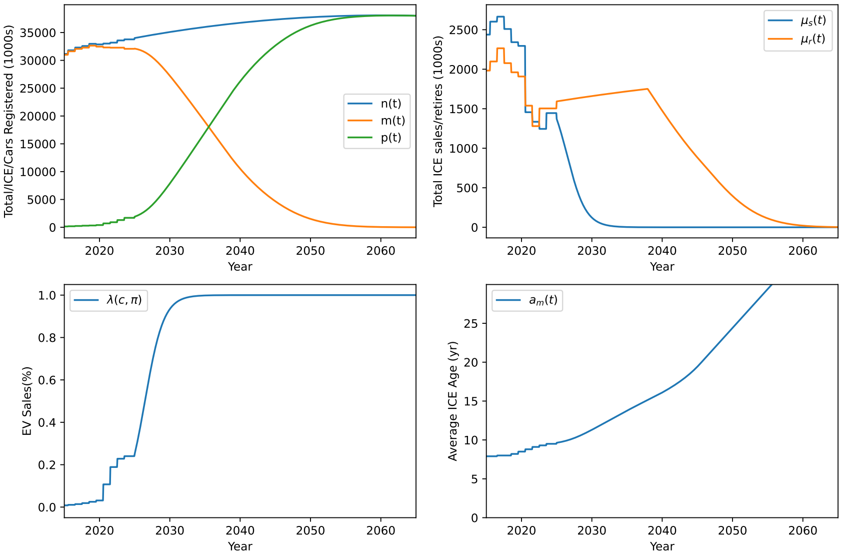

Figure 2: UK case study: a fast transition with sales of EV reaching 100% by 2035.

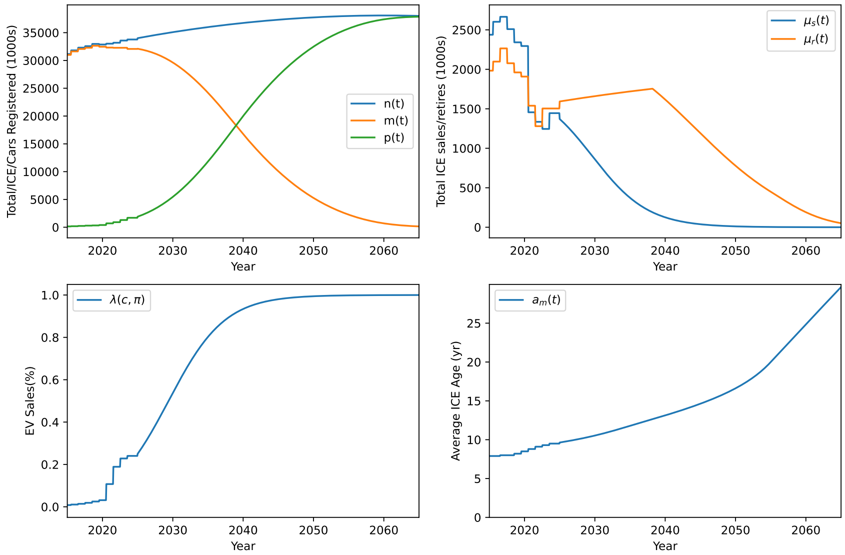

Figure 3: UK case study: a slow transition with sales of EV only reaching 100% by 2050.

penalties associated with this scenario are large, we expect the optimal policy would use a fast transition supported by incentives to be the most cost effective way to manage the system.

As these initial results show, we are confident that we are able to predict how a changes in the number of sales now will affect the number of cars retiring up to 15 or 20 years in the future, using only observations of the state of the world at the current time. This will be essential when it comes to solving the problem numerically, as dynamic programming techniques and other similar methods require that costs and changes in state can be calculated given the current state of the world (i.e. does not depend on historical values). So by including the variables a_n, a_m, and a_p, we have been able to remove any need to keep track of historical sales and retirements, and keep the number of dynamic variables to three.

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Year	UK Pop. (1000s)	No. of Cars (1000s)	Γ (cars per capita)	Sales (1000s)	Retired (1000s)	Average Age (years)
2014	64620	30871	0.48			7.9
2015	65088	31154	0.49	2661	1983	7.9
2016	65607	31814	0.48	2723	2098	7.9
2017	65966	32303	0.50	2564	2265	8.0
2018	66289	32578	0.49	2394	2077	8.0
2019	66631	32973	0.49	2347	1961	8.2
2020	66744	32869	0.49	1656	1908	8.5
2021	66984	33019	0.49	1677	1539	8.8
2022	67602	33187	0.49	1652	1280	9.1
2023	68265	33582	0.49	1946	1504	9.3

Table 1: Using data from Department for Transport (2024) we construct the number of sales and retirements per year.

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