

An error of temperature-feedback formulism and its consequences

ABSTRACT

Feedback formulism borrowed from control theory is universally misapplied in deriving climate sensitivity. Though in electronic feedback amplifiers the DC input is often small enough to be ignored without error, in climate the overwhelmingly predominant 255 K emission temperature must not be, but was hitherto, omitted from the input to the temperature-feedback loop. In effect, its large feedback response was added to (and miscounted as part of) the actually small feedback response to anthropogenic warming. By that grave control-theoretic error, feedback response was hitherto thought to constitute as much as 2 [1 to 4] K, or 67% [50% to 80%], of the long-projected 3 [2 to 5] K equilibrium doubled-CO₂ sensitivity (ECS). However, at the 1850 equilibrium, feedback response comprised only 7.7% of global temperature, implying only 1.2 K ECS. Allowing for subsequent time-variance, a feedback factor 0.09 would yield 2 KECS, while 0.10 would yield 5 K: but no precision so fine as 0.01 is attainable, vitiating all ECS projections via feedback formulism, hitherto the standard method. After correcting the error, mitigation is unnecessary.

1. Introduction

Feedback formulism in control theory is applicable to all feedback-moderated dynamic systems, from electronic circuits and rockets to climate. Temperature-feedback response, an indirect additional temperature signal engendered by a direct or reference temperature and proportional thereto, was hitherto thought to contribute 67% [50% to 80%] of the projected 3 [2 to 5] K equilibrium sensitivity (ECS) forced by a doubled-CO₂-equivalent anthropogenic enrichment of the atmosphere with radiatively-active gases. Accordingly, the word “feedback” is mentioned 1100 times in IPCC (2013) and 2500 times in IPCC (2021).

The principal temperature feedback process is the water-vapor feedback, “an even more powerful absorber of terrestrial radiation” than CO₂ (Charney 1979). Directly-warmed air may carry more water vapor, itself a radiatively-active gas, amplifying a direct warming. At midrange, all other short-acting and thus sensitivity-relevant feedback intensities (Bates 2016), chiefly the lapse-rate, cloud and albedo intensities, broadly self-cancel (*e.g.*, IPCC 2013, table 9.5). However, the present result requires no knowledge of individual feedback intensities.

Arrhenius (1896, 1906) first sought to allow for water-vapor feedback in deriving ECS, the standard climate-sensitivity metric. However, control theory, the theory of feedback amplification, would not be formalized until Black (1934) and Bode (1945).

2. Initial conditions

Time-subscripts t are **0** *ab initio*; **1** in 1850; **2** at present.

Net inbound flux density Q_0 at top of atmosphere (Eq. 1) is found from 1363.5 W m⁻² total solar irradiance S (DeWitte 2016) and current Earth albedo α , equal to 0.293 (Stephens 2015).

$$Q_0 \equiv S \cdot (1 - \alpha) / 4 = \mathbf{241 \text{ W m}^{-2}}. \quad (1)$$

The Stefan-Boltzmann constant σ (Eq. 2) is the constant of proportionality between Q_0 and emission temperature R_0 (defined next) for π the circumference-diameter ratio, k the Boltzmann constant, h Planck’s constant, and c lightspeed *in vacuo* (Rybicki 1979).

$$\sigma \equiv 2 \cdot \pi^5 \cdot k^4 / (15 \cdot h^3 \cdot c^2) = \mathbf{5.6704 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}}. \quad (2)$$

Emission temperature R_0 is derived in the Stefan-Boltzmann equation (3). So predominant is R_0 that the present result would scarcely be affected even if, after allowing for Hölder's inequalities between nonlinear integrals by integrating emission temperatures derived for all solar zenith angles via Eq. (3), R_0 were appreciably less than shown.

$$R_0 \equiv [Q_0 / \sigma]^{0.25} = \mathbf{255\ K} \Rightarrow dR / dQ = (4 \cdot \sigma \cdot R_0^3)^{-1} = R_0 / (4 \cdot Q_0). \quad (3)$$

The Planck response P_t (Eq. 4), the scaling factor between surface temperature T_t and Q_0 , replaces R_0 by T_t in the derivative $dR/dQ = R_0/(4 \cdot Q_0)$ of Eq. (3). Roe (2009) justifies thus treating P_t as part of the sensitivity reference system, not as a negative feedback. Then the product of P_t and any direct or reference forcing ΔQ_t is a reference sensitivity, while the product of P_t and the net aggregate feedback intensity Λ_t is the unitless feedback factor \mathbf{H}_t .

$$P_2 \equiv T_2 / (4 \cdot Q_0) = 289 / (4 \cdot 241) = \mathbf{0.300\ K\ W^{-1}\ m^2}. \quad (4)$$

In 1850, natural reference sensitivity (NRS) ΔR_1 was 10 K (Schmidt 2010).

Reference temperature R_1 was the **265 K** sum of the 255 K R_0 and 10 K NRS ΔR_1 .

The open-loop gain factor or direct-gain factor \mathbf{G}_1 was the ratio **1.039** of R_1 to R_0 .

The 288 K equilibrium temperature E_1 was equal to $\mathbf{G}_1 / (R_0 + F_1)$, where –

The 22.1 K feedback response F_1 , equal to $(E_1 - R_1) / \mathbf{G}_1$, was only 7.7% of E_1 .

At present, doubled-CO₂ forcing ΔQ_2 (CMIP6 mean: Zelinka 2020) is **3.52 W m⁻²**.

The 1.052 K reference doubled-CO₂ sensitivity (RCS) ΔR_2 is equal to $\Delta Q_2 / P_2$.

The 266 K reference temperature R_2 is the sum of R_1 and ~1 K RCS ΔR_2 , so that –

The closed-loop gain factor (or direct-gain factor) \mathbf{G}_2 is the ratio **1.043** of R_2 to R_0 .

Feedback variables are not *italicized*. Normative variables $\Lambda_2, \mathbf{H}_2, A_2, F_2$ (formulism a) and $\Lambda_2^*, \mathbf{H}_2^*, A_2^*, F_2^*$ (b) are UPPERCASE. Defective variables $\lambda_2, \mathbf{h}_2, a_2, \Delta f_2$ (c) are lowercase.

Net aggregate feedback intensity Λ_2 , the originating feedback variable, in W m⁻² per Kelvin of present-day reference temperature R_2 (or of equilibrium temperature E_2) is the sum of the short-acting, sensitivity-relevant feedback intensities (IPCC 2021, table 7.10). Here, however, no knowledge of individual feedback intensities is needed.

The feedback factor \mathbf{H}_2 (unitless) is equal to $\Lambda_2 \cdot P_2$ and to $(E_2 - R_2) / (E_2 \cdot \mathbf{G}_2)$.

The closed-loop gain factor A_2 (unitless) is equal to E_2 / R_0 and to $\mathbf{G}_2 / (1 - \mathbf{G}_2 \cdot \mathbf{H}_2)$.

Feedback response F_2 (in Kelvin) is equal to \mathbf{H}_2 / E_2 and to $(E_2 - R_2) / \mathbf{G}_2$.

Three feedback formulisms will be considered: (a) the normative control-theoretic formulism; (b) a simplified normative formulism; and (c) the defective formulism used in climatology.

3. The error of control-theoretic physics in climatology

Present-day reference temperature R_2 is the 266 K sum of 255 K emission temperature R_0 (Eqs. 1–3) and two direct-gain or *perturbation* signals: the 10 K NRS ΔR_1 and the 1 K RCS ΔR_2 . Net aggregate feedback intensity Λ_2 , the originating temperature-feedback variable in today's climate, necessarily acts in response to the entire reference temperature R_2 , and in close proportion to the amplitude of each of the components therein.

By a grave error of control-theoretic physics, however, all sensitivity studies omit the input signal R_0 from the temperature-feedback loop, though R_0 exceeds all other temperature signals by two orders of magnitude. They also omit the 8 K natural-gain signal, NRS ΔR_1 , which exceeds the 1 K anthropogenic direct-gain signal, RCS ΔR_2 by an order of magnitude. Instead, they treat RCS as though it were the input signal to the temperature-feedback loop.

In effect, then, the feedback responses F_0 to emission temperature R_0 and Δf_1 to NRS ΔR_1 are added to, and miscounted as part of, the actually small feedback response Δf_2 to RCS ΔR_2 . Thus, Hansen (1984), the earliest authority explicitly to deploy control theory in deriving feedback response and hence ECS, introduced the control-theoretic error by taking RCS ΔR_2 as 1.2–1.3 K and ECS ΔE_2 as 4 K. Feedback variables were treated as though they responded solely to RCS, with the implication that the defective closed-loop-gain factor a_2 [misidentified *ibid.* as the “feedback factor”] was $\Delta E_2 / \Delta R_2$, the ratio of ECS to RCS.

“Our ... model yields a warming of ~ 4 C for doubled CO_2 . This indicates a net [defective closed-loop gain factor a_2] of ... 3–4.”

Yet the normative closed-loop gain factor A_2 was 1.129, implying < 1.2 K ECS (Eq. 5).

$$A_2 \equiv E_2 / R_0 = 288/255 \Rightarrow \Delta E_2 \equiv \Delta R_2 \cdot A_2 = 1.052 \cdot 1.129 < \mathbf{1.2\ K.} \quad (5)$$

Likewise, IPCC (2021, p. 2222) [*author’s emphases*] misdefined “climate feedback” as –

“An interaction in which a *perturbation* in one climate quantity causes a *change* in a second, and the *change* in the second ultimately leads to an additional *change* in the first. A negative feedback is one in which the initial *perturbation* is weakened by the *changes* it causes; a positive feedback is one in which the initial *perturbation* is enhanced. The initial *perturbation* can either be externally forced or arise as part of internal variability.”

Incomplete definitions such as these, and the resultant error, are universal in climatology (e.g. Hansen 1984; Schlesinger 1988; IPCC 1990–2021; Ruddiman 2001; Bony 2006; Soden 2006; Bates 2007, Roe 2009; Goosse 2010; Lacis 2010, 2013; Schmidt 2010; Lindzen 2011; Knutti 2015; Dufresne 2015; Prentice 2015; Sherwood 2015, 2020; Ye 2015; Baboo 2018; Heinze 2019; AMS 2020; Schmittner 2021; Visconti 2023; FutureLearn 2024; UCAR 2025).

Interdisciplinary compartmentalization has inhibited detection of the error, by which the defective feedback response Δf_2 to 1 K RCS ΔR_2 was thought to fall on 2 [1 to 4] K, representing 67% [50% to 80%] of projected ECS ΔE_2 on 3 [2 to 5] K (e.g., Charney 1979; IPCC *passim*), implying a defective feedback factor h_2 falling on the interval 0.67 [0.5 to 0.8].

4. Theory

a. Formulism (a): The normative control-theoretic temperature-feedback amplifier

The equations shown in the normative (a) temperature-feedback amplifier block diagram (Fig. 1) obtain at the operating point at time $t = 2$ once the short-acting feedbacks have acted and temperature has resettled to equilibrium. The input signal, the 255 K emission temperature R_0 , enters the feedback loop at the summative (+) input node, whence the sum of R_0 and the feedback factor F_t is sent to the direct-gain block. There, F_t is multiplied by the open-loop gain factor G_t , the ratio of R_t to R_0 (Eq. 6), to become the output signal, equilibrium temperature E_t (Eq. 7), itself the product of R_0 and the closed-loop gain factor A_t (Eq. 8). Next, the signal passes to the feedback block, where it is multiplied by the feedback factor H_t (Eq. 9), itself the product of feedback intensity Λ_t and the Planck response P_t , to form the feedback response F_t (Eq. 10).

NB: Here, for simplicity and clarity, a net-positive feedback intensity and feedback factor is taken as implying a net-positive feedback response.

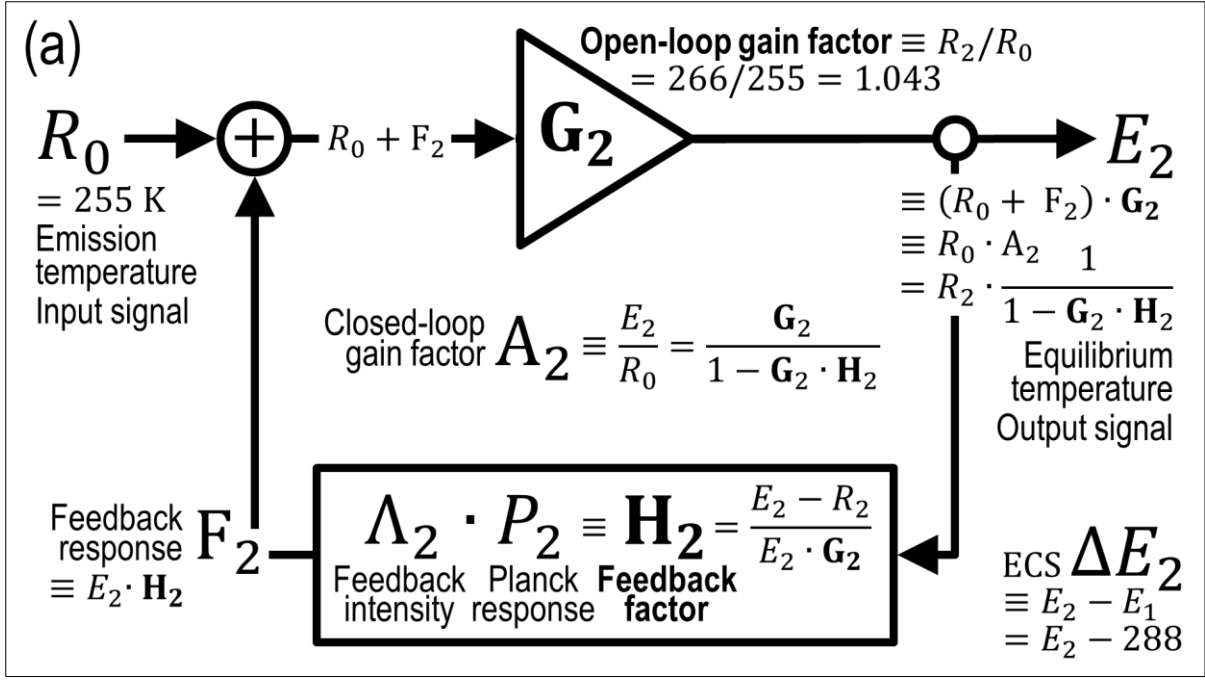


Fig. 1. Normative control-theoretic temperature-feedback amplifier (a) at time $t = 2$.

Eqs. (6–10), then, are the definitional equations of the temperature-feedback amplifier.

$$\mathbf{G}_t \equiv R_t / R_0; \quad (6)$$

$$E_t \equiv (R_0 + F_t) \cdot \mathbf{G}_t; \quad (7)$$

$$A_t \equiv E_t / R_0; \quad (8)$$

$$\mathbf{H}_t \equiv \Lambda_t \cdot P_t; \quad (9)$$

$$F_t \equiv E_t \cdot \mathbf{H}_t. \quad (10)$$

The following expressions for $F_t, \mathbf{H}_t, A_t, E_t$ (Eqs. 11–14), derived from the above definitional equations (6–10), are relevant hereto:

$$\begin{aligned} E_t &\equiv (R_0 + F_t) \cdot \mathbf{G}_t = R_t + F_t \cdot \mathbf{G}_t \\ \Rightarrow F_t &= (E_t - R_t) / \mathbf{G}_t; \end{aligned} \quad (11)$$

$$\begin{aligned} \wedge F_t &\equiv E_t \cdot \mathbf{H}_t \\ \Rightarrow \mathbf{H}_t &= (E_t - R_t) / (E_t \cdot \mathbf{G}_t). \end{aligned} \quad (12)$$

$$\begin{aligned} E_t / \mathbf{G}_t &= R_0 + F_t = R_0 + E_t \cdot \mathbf{H}_t \\ \Rightarrow \mathbf{G}_t \cdot R_0 &= E_t \cdot (1 - \mathbf{G}_t \cdot \mathbf{H}_t); \\ \wedge A_t &\equiv E_t / R_0 \\ \Rightarrow A_t &= \mathbf{G}_t / (1 - \mathbf{G}_t \cdot \mathbf{H}_t). \end{aligned} \quad (13)$$

$$\begin{aligned} E_t &\equiv R_0 \cdot A_t = R_0 \cdot \frac{\mathbf{G}_t}{1 - \mathbf{G}_t \cdot \mathbf{H}_t} = R_t \cdot \frac{1}{1 - \mathbf{G}_t \cdot \mathbf{H}_t} = \sum_{k=0}^{\infty} R_t \cdot (\mathbf{G}_t \cdot \mathbf{H}_t)^k \\ &= R_t \cdot (\mathbf{G}_t \cdot \mathbf{H}_t)^0 + R_t \cdot (\mathbf{G}_t \cdot \mathbf{H}_t)^1 + R_t \cdot (\mathbf{G}_t \cdot \mathbf{H}_t)^2 + \dots + R_t \cdot (\mathbf{G}_t \cdot \mathbf{H}_t)^{\infty}. \end{aligned} \quad (14)$$

Equilibrium temperature E_t (Eq. 14) is the sum of an infinite Archimedean geometric progression of first term R_t and common ratio $\mathbf{G}_t \cdot \mathbf{H}_t$, the product of the direct-gain and feedback-gain factors, subject to the convergence criterion $(\mathbf{G}_t \cdot \mathbf{H}_t < 1) \in \mathbb{R}$, which is amply satisfied given that emission temperature R_0 predominates in reference temperature R_t . As the temperature signal circulates infinitely around the feedback loop, on each pass through the gain and feedback blocks the signal is incremented by the product of R_t and a successively greater power of $\mathbf{G}_t \cdot \mathbf{H}_t$. Today, on each pass $\mathbf{G}_2 \cdot \mathbf{H}_2$ acts on R_2 , the sum of $R_0, \Delta R_1$ and ΔR_2 .

b. Formulism (b): The simplified normative control-theoretic feedback amplifier*

Since the open-loop gain factor \mathbf{G}_2 in the normative feedback formulism (a) (Fig. 1) barely exceeds unity, with little error one may omit the \mathbf{G}_2 gain block and replace the input signal R_0 by the 264 K reference temperature R_2 (Fig. 2b). Since \mathbf{G}_2 is omitted in climatology [or sub-optimally replaced with a forcing-to-temperature converter, and then with a temperature-to-forcing back-converter in the feedback block], the simplified normative formulism (b) (Fig. 2b) facilitates direct comparison with the defective climatological formulism (c) (Fig. 2c). Feedback parameters in formulism (b) are indicated by asterisks. Equilibrium temperature E_2 becomes the sum of R_2 and the feedback response F_2^* , for F_2^* the product of E_2 and the feedback factor \mathbf{H}_2^* , itself the product of feedback intensity Λ_2^* and the Planck response P_2 ; and the closed-loop gain factor A_2^* , the ratio of E_2 to R_2 , is also the reciprocal of $(1 - \mathbf{H}_2^*)$.

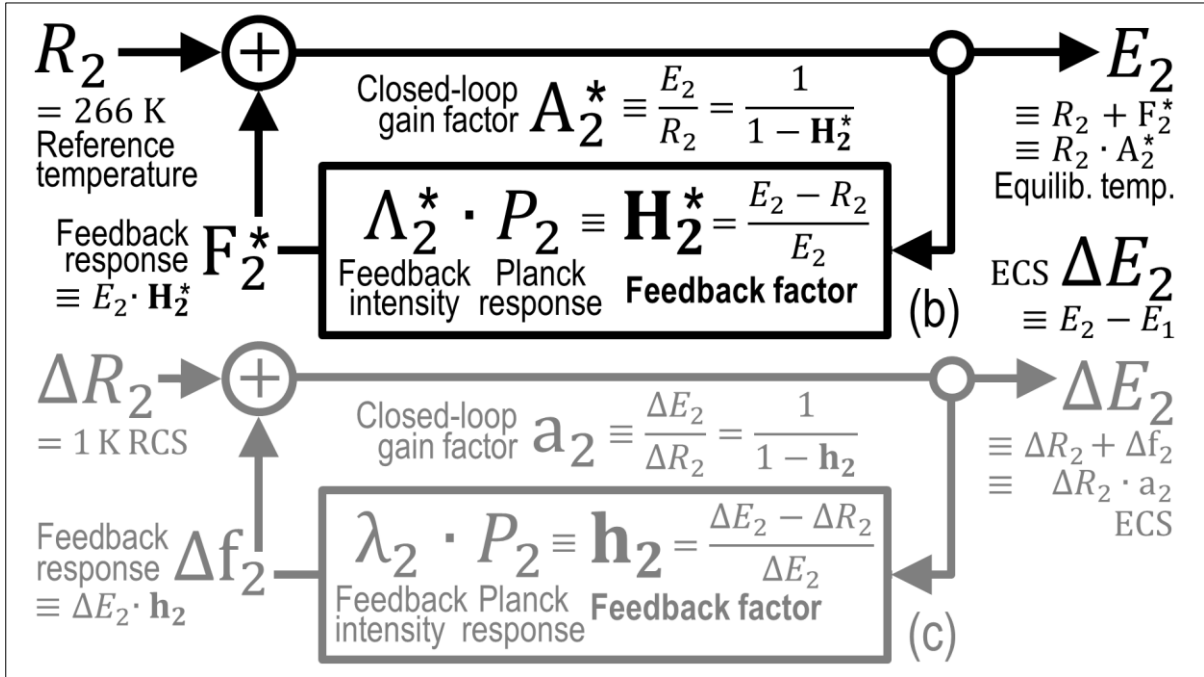


Fig. 2. (b) Simplified* control-theoretic feedback amplifier; (c) Defective feedback amplifier.

c. Formulism (c): The defective feedback amplifier universal in climatology

In climatology, the 1 K RCS ΔR_2 , a gain signal, is taken as the sole input to the feedback loop (Fig. 2c). Thus, the \mathbf{G}_2 gain block is omitted (or replaced by a converter with a forcing ΔQ_2 as the input). Erroneously, neither the 255 K emission temperature R_0 nor the 10 K natural reference sensitivity ΔR_1 is represented in the defective temperature-feedback loop. Instead, deltas replace absolute quantities: ECS ΔE_2 replaces the output signal, equilibrium temperature E_2 ; the product \mathbf{h}_2 of defective feedback intensity λ_2 and the Planck response P_2 replaces the feedback factor \mathbf{H}_2^* ; Δf_2 , the product of ΔE_2 and \mathbf{h}_2 , replaces the feedback response F_2^* ; and a_2 , the reciprocal of $(1 - \mathbf{h}_2)$, replaces the closed-loop gain factor A_2^* .

In control theory the DC input signal is often neglected where it is small compared with the AC gain and feedback signals. However, in climate the input signal predominates and must be kept.

Currently, the feedback responses F_0 to the input signal (emission temperature R_0) and ΔF_1 to the natural direct-gain signal (NRS ΔR_1), together comprising $> 99.5\%$ of total feedback response, are omitted and are, in effect, miscounted as though they were part of the actually small feedback response ΔF_2 to RCS ΔR_2 ; and linearization about the present state of the climate is then unsuccessfully attempted by way of a first-order Taylor-series expansion.

Since defective λ_2 and \mathbf{h}_2 are excessive by an order of magnitude, the defective closed-loop gain factor a_2 (and hence the defective ECS ΔE_2) is excessive by a factor 2–5.

5. Results

a. Feedback variables implicit in currently-projected ECS

Table 1 derives feedback variables (Eqs. 16abc–19abc) implicit in projected [2 to 5] K ECS. The interval of Λ_2 that would yield [2 to 5] K ECS is 0.27 [0.26 to 0.29] $\text{W m}^{-2} \text{K}^{-1}$ (Eq. 18a), requiring derivation of Λ_2 to an unattainable 0.01 $\text{W m}^{-2} \text{K}^{-1}$ precision.

ΔE_2	ECS	2 K	3 K	4 K	5 K	(IPCC 2021)	(Eq.)
$E_2 \equiv$	$E_1 + \Delta E_2$	290 K	291 K	292 K	293 K	Equilibr. temp.	(15)
a) $A_2 \equiv$	E_2 / R_0	1.14	1.14	1.15	1.15	Closed-loop gain factor (unitless)	(16a)
b) $A_2^* \equiv$	E_2 / R_2	1.09	1.09	1.10	1.10		(16b)
c) $a_2 \equiv$	$\Delta E_2 / \Delta R_2$	2.00	3.00	4.00	5.00		(16c)
a) $\mathbf{H}_2 \equiv$	$(E_2 - R_2) / (E_2 \cdot \mathbf{G}_2)$	0.08	0.08	0.09	0.09	Feedback factor (unitless)	(17a)
b) $\mathbf{H}_2^* \equiv$	$(E_2 - R_2) / E_2$	0.08	0.09	0.09	0.09		(17b)
c) $\mathbf{h}_2 \equiv$	$(\Delta E_2 - \Delta R_2) / \Delta E_2$	0.50	0.67	0.75	0.80		(17c)
a) $\Lambda_2 \equiv$	\mathbf{H}_2 / P_2	0.26	0.27	0.28	0.29	Feedback intensity ($\text{W m}^{-2} \text{K}^{-1}$)	(18a)
b) $\Lambda_2^* \equiv$	\mathbf{H}_2^* / P_2	0.28	0.29	0.30	0.31		(18b)
c) $\lambda_2 \equiv$	\mathbf{h}_2 / P_2	1.58	2.17	2.46	2.63		(18c)
λ_2	<i>IPCC(2021, table 7.10)</i>	-1.81	-1.16	—	-0.51		(19d)
$+ P_2^{-1} $	<i>ibid.</i>	+3.40	+3.22	—	+3.00		(19e)
$= \lambda_2$	<i>cf. Eq. (18c)</i>	1.59	2.06	—	2.49		(19f)
	$\mathbf{G}_2 \cdot P_2 \cdot R_2 / (1 - \mathbf{G}_2 \cdot P_2 \cdot \Lambda_2)^2$	99	100	100	101	Derivative ($\frac{\text{K}}{\text{W m}^{-2} \text{K}^{-1}}$)	(20a)
$\frac{d(\Delta E)}{d\Lambda}$	$P_2 \cdot R_2 / (1 - P_2 \cdot \Lambda_2^*)^2$	95	95	96	97		(20b)
	$P_2 \cdot \Delta R_2 / (1 - P_2 \cdot \lambda_2)^2$	1.1	2.6	4.6	7.1		(20c)

Table 1. Results by a) the normative; b) the simplified* normative; c) the defective method

The derivatives of the system-response curve of ECS ΔE_2 on [2 to 5] K against net aggregate feedback intensity are obtained in Eqs. 20abc. The derivative $d(\Delta E)/d\Lambda$ (Eqs. 20ab) of the normative system-response curve is of order 100 K per unit of feedback intensity Λ_2 , demonstrating the extent to which temperature is hypersensitive to time-variance in Λ_2 .

In Eqs. (21–22), feedback intensities are derived directly from projected ECS and *vice versa*.

Feedback intensities $\Lambda_2, \Lambda_2^*, \lambda_2$ implicit in projected ECS ΔE_2	$\Lambda_2 = \left(\frac{1}{\mathbf{G}_2} - \frac{R_0}{E_1 + \Delta E_2} \right) / P_2 = \left(\frac{255}{266} - \frac{255}{288 + \Delta E_2} \right) / 0.3. \quad (21a)$
	$\Lambda_2^* \approx \left(1 - \frac{R_2}{E_1 + \Delta E_2} \right) / P_2 \approx \left(1 - \frac{266}{288 + \Delta E_2} \right) / 0.3. \quad (21b)$
	$\lambda_2 = (1 - \Delta R_2 / \Delta E_2) / P_2 = (1 - 1 / \Delta E_2) / 0.3. \quad (21c)$
ECS ΔE_2 derived from feedback intensities $\Lambda_2, \Lambda_2^*, \lambda_2$	$\Delta E_2 = \frac{R_2}{1 - \mathbf{G}_2 \cdot P_2 \cdot \Lambda_2} - E_1 = \frac{266}{1 - 1.0353 \cdot 0.3 \cdot \Lambda_2} - 288. \quad (22a)$
	$\Delta E_2 \approx R_2 / (1 - P_2 \cdot \Lambda_2^*) - E_1 \approx 266 / (1 - 0.3 \cdot \Lambda_2^*) - 288. \quad (22b)$
	$\Delta E_2 = \Delta R_2 / (1 - P_2 \cdot \lambda_2) = 1 / (1 - 0.3 \cdot \lambda_2). \quad (22c)$

b. The response curve of ECS against normative Λ_2 (Fig. 3a)

The response curve of ECS against normative Λ_2 on [0.26 to 0.29] $\text{W m}^{-2} \text{K}^{-1}$, of interval breadth only $0.03 \text{ W m}^{-2} \text{K}^{-1}$ (Fig. 3a; Eq. 18a), is very close to linear. Visibly, constraint of ECS via derivation of Λ_2 is untenable, particularly given the $1.3 \text{ W m}^{-2} \text{K}^{-1}$ breadth of the published interval of uncertainty in net aggregate feedback intensity (IPCC 2021, table 7.10).

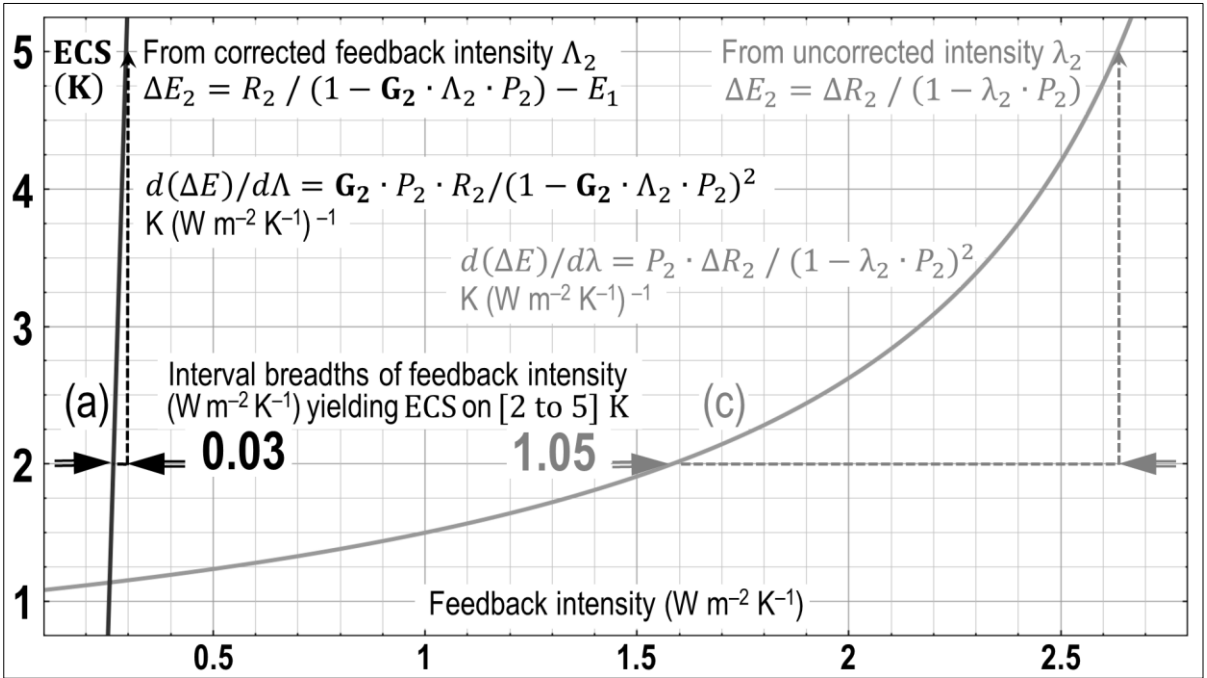


Figure 3. Response curve of ECS against (a) normative and (c) defective feedback intensities.

c. The response curve of ECS against defective λ_2 (Eqs.18c, 22c)

The defective response curve of ECS against λ_2 on [1.58 to 2.63] $\text{W m}^{-2} \text{K}^{-1}$ (Eq. 18c) is rectangular-hyperbolic: the derivative $d(\Delta E)/d\lambda$ (Eq. 20c) rapidly steepens with λ_2 (Fig. 3c). Therefore, even if it were acceptable to overlook the control-theoretic error, the defective feedback formulism cannot constrain ECS without an unattainably precise knowledge of λ_2 . To illustrate the amplitude of the error, if defective λ_2 were input to the normative system-response equation (22a), ECS would fall not on [2 to 5] K but on [239 to 1223] K.

In electronic feedback circuits, where the AC open-loop gain and feedback signals often exceed the small DC input signal by orders of magnitude, the input signal may be discarded with little error. In climate, however, there exists no differencer permitting the feedback factor \mathbf{H}_2 to respond more to RCS ΔR_2 than to NRS ΔR_1 or to emission temperature R_0 . Inanimate feedback processes cannot distinguish between 1 K of RCS, of NRS or of emission temperature. Since $\mathbf{G}_2 \approx 1$, at any time t they must respond in close proportion to each Kelvin of each of the three components in the entire reference temperature R_2 .

For this reason, in temperature-feedback amplification it is impermissible to neglect any of the three components in R_2 . In particular, the 255 K emission temperature R_0 erroneously omitted from the input signal to the temperature-feedback loop exceeds the input signal RCS ΔR_2 in the defective feedback formulism (c) by 2 orders of magnitude, so that R_0 cannot be omitted from the input to the temperature-feedback loop without large error. So predominant is R_0 that, where it is omitted from the input to the temperature-feedback loop, attempted linearization around the present state of the climate becomes meaningless.

d. Propagation of uncertainty in general-circulation models' time-step algorithms

Where R_0 is transmitted into the feedback loop (Figs. 1, 2b), the hypersensitivity of ECS even to minuscule time-variance in normative feedback intensity Λ_2 compounds an established defect in the general-circulation models. The interdisciplinary divide between climatology and statistics has inhibited modelers from taking due account of propagation of uncertainty in through the models' typically hourly time-steps, over decades to centuries, in just one initial condition, the global cloud fraction, renders any ECS projection falling within a ± 15 K uncertainty envelope merely speculative (Frank 2019). All current projections by diagnosis of feedback variables from models' outputs (*e.g.*, Vial 2013) fall within that broad envelope of uncertainty. Since all such projections are further compromised by the control-theoretic error, they are valueless *a fortiori*. Since feedback formulisms (a), (b), (c) cannot reliably constrain ECS, other methods, rooted in observation rather than in general-circulation models' outputs sent to a feedback-amplifier algorithm, must be deployed.

e. Observational derivations of ECS

Four observational methods independent of feedback formulism cohere in yielding ECS on [1 to 2.5] K rather than on the [2 to 5] K *e.g.*, in Charney (1979) and IPCC (*passim*) –

a) The energy-budget method (Gregory 2004; Lewis & Curry 2014, 2018) depends not on feedback formulism but on observation. It generates lesser ECS values than the projections overstated in consequence of the control-theoretic error: at midrange, typically < 2 K.

b) *A priori*, the many thermostatic processes in climate (such as ocean heat capacity, thermal inertia of ice or earlier tropical afternoon convection with warming) render it le Châtelier-unlikely that feedback intensity Λ_t is time-variant in the industrial era, particularly since the anthropogenic perturbation RCS is small, in which event ECS is as derived in Eq. (5) from the data for the 1850 equilibrium: *i.e.*, < 1.2 K.

c) Anthropogenic forcing by radiatively-active gases since 1850 is 3.2 W m^{-2} (NOAA 2024), coincidentally close to projected effective doubled- CO_2 forcing (ERF) ΔQ_2 . Observed warming since 1700 is of order 1.6 K (Jarvis 2024). Assuming no unrealized warming from pre-existing emissions (significant unrealized sub-centennial-scale warming being unlikely after correcting the control-theoretic error), ECS is **1.6 K**.

d) Though $0.3 [0.2 \text{ to } 0.5] \text{ K decade}^{-1}$ transient warming and $3 [2 \text{ to } 5] \text{ K ECS}$ are projected (Charney 1979; IPCC 1990, 2021), since 1979 only $0.15\text{--}0.25 \text{ K decade}^{-1}$ is observed (Spencer 2025; Morice 2012, 2021). Thus, *pro rata*, ECS falls on **1.5–2.5 K**.

f. Verifications

a) Test apparatus: John Whitfield, a control engineer, constructed a feedback amplifier to emulate temperature feedback and confirm that feedbacks must respond to the input signal (also known in electronics as the setpoint) as well as to any subsequent perturbations thereof.

b) A national laboratory of physics thereupon constructed its own apparatus, with which it conducted 23 experiments confirming that feedback processes must respond to any sufficiently substantial input signal – a fact that is in any event inherent in the control-theoretic equations governing feedback amplifiers.

c) The absent ‘hot spot’: Models misrepresent the altitudinal profile of water-vapor feedback. Though by the Clausius-Clapeyron relation the air may carry $7\% \text{ K}^{-1}$ more water vapor at industrial-era temperatures (Wentz 2007), specific humidity is rising at that rate only in the boundary layer (Kalnay 1996), where, however, non-radiative transports limit its influence and its spectral lines are near saturation. As humidity increases, only the far wings add to infrared absorption (Harde 2017), which in any event varies quasi-logarithmically with specific humidity. In the mid-troposphere, specific humidity has declined since 1948 (Kalnay *op. cit.*), while nearly all general-circulation models project that warming should increase it.

Though models thus project a “hot spot” at 200–300 mb in the tropics warming at twice the surface rate (*e.g.*, IPCC 2007, fig. 9.1c; Lee 2007), declining specific humidity at that pressure altitude (Kalnay *op. cit.*; Paltridge 2009) explains its absence from nearly all datasets (*e.g.*, Lanzante 2006, fig. 5.7E) and explains a mid-troposphere warming rate 30% of models’ mean projection (Spencer 2025). The hot spot being absent, the water-vapor feedback is well below models’ current projections, coherent with the present result.

6. Discussion

Emission temperature, the true input signal to the temperature-feedback loop, exceeds RCS, the defective input signal, by two orders of magnitude. Feedback intensity acts in response not only to the anthropogenic direct-gain signal RCS (as was hitherto thought) but also to the natural direct-gain signal NRS and, above all, to the predominant emission temperature.

After correction, the slope of the system-response curve of ECS against feedback intensity may readily be derived by multivariate calculus, as in Eqs. 20ab. Since the normative response curve (Fig. 3a) is already close to linear, linearization by way of a first-order Taylor-series expansion is unnecessary. The difficulty lies not in deriving the slope of the curve at the point corresponding to a given value of net aggregate feedback intensity but rather, given the large uncertainties in the underlying data and the hypersensitivity of global temperature to time-variance in net aggregate feedback intensity, in constraining that feedback intensity itself.

Though the interval breadth of projected feedback intensities is $1.3 \text{ W m}^{-2} \text{ K}^{-1}$ (IPCC 2021, table 7.10), the normative feedback intensity for 1850 (the ratio of the feedback factor 0.077 to the then Planck response $0.299 \text{ K W}^{-1} \text{ m}^2$) was $0.257 \text{ W m}^{-2} \text{ K}^{-1}$; and the interval that would yield the long-projected $[2 \text{ to } 5] \text{ K ECS}$ today is $[0.264 \text{ to } 0.294] \text{ W m}^{-2} \text{ K}^{-1}$.

Even the normative feedback formulism, then, cannot constrain ECS due to the small amplitude, narrow interval, observational immensurability, large uncertainty and unknown time-variance of net aggregate feedback intensity.

Since feedback processes act on a reference signal greater by orders of magnitude than the 1 K RCS hitherto adopted as the input signal to the feedback loop, equilibrium temperature is hypersensitive even to very small time-variance in feedback intensity: for each $1 \text{ W m}^{-2} \text{ K}^{-1}$ increase in feedback intensity, ECS would be expected to increase by $\sim 100 \text{ K}$. However, since feedback intensity is proving time-invariant in the industrial era, as is to be expected in a near-thermostatic system, the hypersensitivity of temperature to feedback intensity is not proving problematic. Observationally-derived midrange feedback response since 1850 is approximately half of RCS and thus one-third of ECS, not two-thirds as was hitherto thought.

The 3 K projected ECS interval breadth has not changed since Charney (1979) and IPCC (1990): but fear of “tipping points” is an artefact of the error, arising from the shape of the defective rectangular-hyperbolic system-response curve across the interval of overstated ECS projections (Fig. 3c), and is contradicted by the observed near-thermostasis of the climate system in the industrial era. Moreover, where significant time-variance in feedback intensity is assumed, the linearity of observed temperature response to CO_2 forcing (Jarvis 2024) is inconsistent with the defective formulism (c) (Fig. 3c).

Due to the error, feedback intensity was hitherto overstated by an order of magnitude. Consequently, projected [2 to 5] K ECS was overstated by a factor 2. Methods independent of feedback formulism, and thus free of the control-theoretic error, yield only 1–2.5 K warming to 2100. Upon correction, climate risk is significantly reduced, since dangerous warming ceases to be near-certain and becomes ever less likely as transient global warming continues at a net-beneficial rate little more than half of the long-standing $0.3 \text{ K decade}^{-1}$ midrange projection that has prevailed since IPCC 1990.

7. Conclusion

Correcting the control-theoretic error in climatology by which the predominant emission temperature was universally omitted from the input to the climate-feedback loop removes the apparently overriding, error-driven imperative to replace coal, oil and gas with far costlier and less reliable (and, in many Western nations, already wastefully overbuilt) wind and solar generation, which must in any event be backed up at all times by thermal generation at wasteful, continuously-spinning reserve.

While coal, oil and gas reserves endure, the West may safely retain thermal generation for competitiveness, energy security and affordability, even as China and Russia, India and Pakistan rapidly expand it in the East (Shaw 2023; Chaganti Singh 2023; APP 2023; Statista 2023). Adaptation (if needed) is the rational economic choice; and, given the growing security as well as terms-of-trade threats posed by the West’s near-unilateral transition to net zero, it is also the strategic imperative.

After correction of the control-theoretic error herein identified, mitigation inexpensive enough to be affordable will be ineffective, especially if it remains near-unilateral, while mitigation expensive enough to be effective will be as unaffordable as it is now unnecessary.

Acknowledgements.

The author thanks Professor Matt Briggs, Mr Alex Henney, Professor Dietrich Jeschke, Professor David Legates, Dipl.-Ing. Michael Limburg, Mr William Rostron, Dr Thomas Sheahan, Dr Willie Soon and Mr John Whitfield for research contributions; Mr William Bailey, Professor Freeman Dyson, Professor Chris Essex, Professor Dr Hermann Harde, Professor Dr Mojib Latif, Mr Nic Lewis, Dr Benoît Rittaud, Academician Dr Vladimir Semenov, Professor Nir Shaviv, Dr Roger Taguchi, Mr George White and Academician Dr Nina Zaytseva for discussions, Professor Ray Bates, Professor John Dewey, Baron Clanmorris of Newbrook, Professor Will Happer, Dr David Heald, Professor Jose Rodal, Professor Murry Salby and Dr Jack Sarfatti for pre-submission reviews; Mr Hal Shurtleff, Mr G. Edward Griffin, Ms Pamela Matlack-Klein, Professor Maria da Assunção Ferreira Pedrosa de Araújo, Professor Nils-Axel Mörner, Andrew Bridgen MP, the City Government of Moscow, the Heartland Institute, the Deutscher Bundestag, the Europäisches Institut für Klima und Energie, the Climate Intelligence Group and the Parliament of the Czech Republic for facilitating discussion at international conferences; and Professor Antonino Zichichi for having provided, at the Centre for Scientific Culture in Erice, Sicily, the high-level scientific forum from which these ideas sprang.

Data Availability Statement.

Supplemental matter providing a history of the control-theoretic error and a national laboratory's report confirming the present result is available from the author.

REFERENCES

- AMS, 2020: *Glossary of Meteorology*. American Meteorological Society, Boston.
- APP, 2023: Govt plans to switch to coal to provide citizens reliable electricity. *Express Tribune*, Karachi, February 15.
<https://tribune.com.pk/story/2401260/govt-plans-to-quadruple-domestic-coal-fired-energy#:~:text=Pakistan%20plans%20to%20quadruple%20its%20domestic%20coal-fired%20capacity,it%20seeks%20to%20ease%20a%20foreign%20exchange%20crisis.>
- Arrhenius, S., 1896: On the influence of carbonic acid in the air upon the temperature of the ground. *Edinburgh & Dublin J. Philosoph. Mag. & J. Sci.*, **41** (251), 237–276.
- Arrhenius, S., 1906: Die vermutliche Ursache der Klimaschwankungen (“The possible cause for climate variability”). *Meddelanden från K. Vetenskapsakademiens Nobelinstitut*, **1** (2), 1ff.
- Baboo, P., 2018, *How water vapors regulate global warming*, Institution of Engineers, Kolkata, India.
[https://www.researchgate.net/post/How_water_vapors_regulate_the_global_warming.](https://www.researchgate.net/post/How_water_vapors_regulate_the_global_warming)
- Bates, J.R., 2007: Some considerations of the concept of climate feedback, *Q. J. R. Met. Soc.* **133** (624), 545–560, <https://doi.org/10.1002/qj.62>.
- Bates, J.R., 2016: Estimating climate sensitivity using two-zone energy-balance models. *Earth Space Sci.* **3** (5), 207–225. <https://doi.org/10.1002/2015EA000154>.
- Black, H.S., 1934: Stabilized feedback amplifiers. *Bell Syst. Tech. J.* **13** (1), 1–18,
<https://doi.org/10.1002/j.1538-7305.1934.tb00652>.
- Bode, H.W. 1945: *Network analysis and feedback amplifier design*. Van Nostrand Reinhold, New York, USA.
- Bony, S., and Coauthors, 2006: How well do we understand and evaluate climate change feedback processes? *J. Clim.* **19**, 3445–3482, <https://doi.org/10.1175/JCLI3819.1>.

- Chaganti Singh, S., 2023: *India scrambles to add coal-fired capacity, avoid outages*. Reuters, London, 29 November.
<https://www.msn.com/en-us/money/markets/exclusive-india-scrambles-to-add-coal-fired-power-capacity-avoid-outages-sources/ar-AA1kKyW5>.
- Charney, J.G., and Coauthors, 1979: *Carbon Dioxide and Climate*. Climate Research Board, Woods Hole, MA, USA.
- DeWitte, S., and S. Nevens, 2016: The total solar irradiance climate data record. *Astrophys. J.*, **830** (25). <https://doi.org/10.3847/0004-637X/830/1/25>.
- Dufresne, J.-L., and M. Saint-Lu, 2015: Positive feedback in climate stabilization or runaway illustrated by a simple experiment. *Bull. Amer. Met. Soc.* 97(5), <https://doi.org/10.1175/BAMS-D-14-00022.1>.
- Frank, P., 2019: Propagation of error and the reliability of global air temperature projections. *Frontiers in Earth Sci.* **7** (223), <https://doi.org/10.3389/feart.2019.00223>.
- FutureLearn, 2024: Radiative Feedbacks. In: *Causes of Climate Change* (online), U. Bergen, <https://www.futurelearn.com/info/courses/causes-of-climate-change/0/steps/13594>.
- Goosse, H., P.Y. Barriat, W. Lefebvre, M.F. Loutre, and V. Zunz, 2010: *Introduction to climate dynamics and climate modeling* (online textbook). Université Catholique de Louvain, https://www.climate.be/textbook/chapter4_node9.html.
- Gregory, J.M., and Coauthors, 2001: A new method for diagnosing radiative forcing and climate sensitivity. *Geophys. Res. Lett.* **31** (3), <https://doi.org/10.1029/2003GL018747>.
- Hansen, J., A. Lacis, D. Rind, and G. Russell, 1984: In: *Climate Processes and Climate Sensitivity (AGU Geophysical Monograph 29)*, Hansen, J., & T. Takahashi, Eds., Amer. Geophys. Union, Washington DC, 130–163, <https://doi.org/10.1029/GM029>.
- Harde, H., 2017: Radiation transfer calculations and assessment of global warming by CO₂. *Int. J. Atmos. Sci.*, <https://doi.org/10.1155/2017/9251034>.
- Heinze, C., and Coauthors, 2019: Climate feedbacks in the Earth system and prospects for evaluation. *Earth Syst. Dyn.* **10** (3), 379–452, <https://doi.org/10.5194/esd-10-379-2019>.
- IPCC, 1990: *Assessment Report prepared for the Intergovernmental Panel on Climate Change by Working Group I*, Houghton, J., et al., Eds. Cambridge Univ. Press.
- IPCC, 2007: *Climate Change 2007: The physical science basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Solomon, S., et al., Eds., Cambridge Univ. Press.
- IPCC, 2013: *Climate change 2013: The physical science basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, Stocker, T.F., et al., Eds., Cambridge Univ. Press.
- IPCC, 2021: *Climate Change 2021: The physical science basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, Masson-Delmotte, V., et al., Eds., Cambridge Univ. Press.
- Jarvis, A., and Forster, P.M., 2024: Estimated human-induced warming from a linear temperature and atmospheric CO₂ relationship. *Nat. Geosci.*, **17**, 1222–1224 (2024), <https://www.nature.com/articles/s41561-024-01580-5>.
- Kalnay, E., and Coauthors, 1996: The NCEP/NCAR Reanalysis 40-year Project. *Bull. Amer. Met. Soc.* **77**, 437–71, [https://doi.org/10.1175/1520-0477\(1996\)077<0437:TNYRP>2.0.CO;2](https://doi.org/10.1175/1520-0477(1996)077<0437:TNYRP>2.0.CO;2).
- Knutti, R., and M.A.A. Rugenstein, 2015: Feedbacks, climate sensitivity and the limits of linear models. *Philosoph. Trans. Royal. Soc. A*, **373**, 20150146.
<http://dx.doi.org/10.1098/rsta.2015.0146>.

- Lacis, A.A., and Coauthors, 2010: Atmospheric CO₂: principal control knob governing Earth's temperature. *Science* **33**, 356–359. <https://doi.org/10.1126/science.1190653>.
- Lacis, A.A., and Coauthors, 2013: The role of long-lived greenhouse gases as principal LW control knob that governs the global surface temperature for past & future climate change. *Tellus B* **65**, 19734. <https://doi.org/10.3402/tellusb.v65i0.19734>.
- Lanzante, J.R., and Coauthors, 2006: CCSP synthesis and assessment report 11. In: *Temperature Trends in the lower atmosphere: steps for understanding and reconciling differences*, Karl, T.R., et al., Eds., National Oceanographic & Atmospheric Administration National Climatic Data Center, Asheville. https://www.gfdl.noaa.gov/jrl_vita_ccsp-sap1-1.
- Lee, M.I., M.J. Suarez, I.S. Kang, I.M. Held, D. Kim, 2007: A moist benchmark calculation for the atmospheric general-circulation models. *J. Clim.* **21**, 4934–4954, <https://doi.org/10.1175/2008JCLI1891.1>.
- Lewis, N., and J.A. Curry, 2014: Implications for climate sensitivity of AR5 forcing and heat uptake estimates. *Clim. Dyn.*, <https://doi.org/10.1007/s00382-014-2342-y>.
- Lewis, N., and J.A. Curry, 2018: The impact of recent forcing and ocean heat uptake data on estimates of climate sensitivity. *J. Clim.* **31**, <https://doi.org/10.1175/JCLI-D-17-0667-s1>.
- Lindzen, R.S., and Y.-S. Choi, 2011: Observational determination of climate sensitivity and its implications. *Asia-Pacific J. Atmos. Sci. B*, **47**, 337–390, <https://doi.org/10.1007/s13143-011-0023-x>.
- Morice, C.P., and Coauthors, 2012: Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: The HadCRUT4 data set. *J. Geophys. Res. (Atmos.)*, <https://doi.org/10.1029/2011JD017187>; www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.6.0.0.monthly_ns_avg.txt.
- Morice, C.P., and Coauthors, 2021: An updated assessment of near-surface temperature change from 1850: the HadCRUT5 dataset. *J. Geophys. Res. (Atmos.)*, <https://doi.org/10.1029/2019JD032361>; https://crudata.uea.ac.uk/cru/data/temperature/HadCRUT5.0Analysis_gl.txt.
- NOAA, 2024. *Annual greenhouse-gas index*, Washington DC, <https://gml.noaa.gov/aggi>.
- Paltridge, G., Arking, A., Pook, M. Trends in middle- and upper-level tropospheric humidity from NCEP reanalysis data. *Theor. Appl. Climatol.* (2009), <https://doi.org/10.1007/s00704-009-0117-x>.
- Prentice, I.C., and Coauthors, 2015: Reliable, robust and realistic: the three R's of next-generation land-surface modelling. *Atmos. Chem. Phys.*, **15**, 5987–6005, <https://doi.org/10.5194/acp-15-5987-2015>.
- Roe, G., 2009: Feedbacks, timescales and seeing red. *Ann. Rev. Earth. Planet. Sci.* **37**, 93–115, <https://doi.org/10.1146/annurev.earth.061008.134734>.
- Ruddiman, W.F., 2001: *Earth's climate, past and future*. W.H. Freeman, New York.
- Rybicki, G., and A. Lightman, 1979: *Radiative Processes in Astrophysics*. John Wiley & Sons, Hoboken, NJ.
- Schlesinger, M.E., 1988: Quantitative analysis of feedbacks in climate model simulations of CO₂-induced warming. In: *Physically-based modelling and simulation of climate and climatic change: NATO ASI Series, Series C, 243*, Schlesinger, M.E., Ed., Springer, Dordrecht, Netherlands, https://doi.org/10.1007/978-94-009-3043-8_2.
- Schmidt, G.A., and Coauthors, 2010: Attribution of the present-day total greenhouse effect. *J. Geophys. Res. (Atmos.)*, **115**, D20106, <https://doi.org/10.1029/2010JD014287>.
- Schmittner, A., 2021. *Introduction to Climate Science*. Oregon State University, <https://open.umn.edu/opentextbooks/textbooks/860>.

- Shaw, A., 2023: China's coal expansion to eclipse renewable energy gains – report. Microsoft Start, 28 November. <https://www.msn.com/en-us/money/companies/china-s-coal-expansion-to-eclipse-renewable-energy-gains-report/ar-AA1kGYiI>.
- Sherwood, S., and Coauthors, 2015: Adjustments in the forcing-feedback framework for understanding climate change. *BAMS* **96** (2), 217–228, <https://doi.org/10.1175/BAMS-D-13-00167>.
- Sherwood, S., and Coauthors, 2020: An assessment of Earth's climate sensitivity using multiple lines of evidence. *Rev. Geophys.* **58** (4), e2019RG000678, <https://doi.org/10.1029/2019RG000678>.
- Soden, B.J., and I.M. Held, 2006: An assessment of climate feedbacks in coupled ocean-atmosphere models. *J. Clim.* **19**, 3354–3360, <https://doi.org/10.1175/JCLI3799.1>.
- Spencer, R., and J. Christy, 2025: UAH monthly global mean lower-troposphere temperature anomalies (University of Alabama in Huntsville), http://www.nsstc.uah.edu/data/msu/v6.0/tlt/uahncdc_lt_6.0.txt.
- Statista, 2023: Distribution of electricity generation in Russia in 2022, by source. <https://www.statista.com/statistics/1237590/russia-distribution-of-electricity-production-by-source>.
- Stephens, G.L., and Coauthors, 2015: The albedo of Earth. *Rev. Geophys.*, **53** (1), 141–63, <https://doi.org/10.1002/2014RG000449>.
- UCAR, 2025: Climate feedback loops and tipping points. Cent. Sci. Educ., Boulder, CO. <https://scied.ucar.edu/learning-zone/earth-system/climate-system/feedback-loops-tipping-points>.
- Vial, J., J.-L. Dufresne, and S. Bony, 2013: Interpretation of inter-model spread in CMIP5 climate sensitivity estimates. *Clim. Dyn.* **41**, 3339–62, <https://doi.org/10.1007/s00382-013-1725-9>.
- Visconti, G., 2023: Greenhouse effect-chemistry climate connection. In: *The fluid environment of the Earth*, Springer, Dordrecht, https://doi.org/10.1007/978-3-031-31539-8_9.
- Wentz, F.J., and Coauthors, 2007: How much more rain will global warming bring? *Science* **317**, 233–35, <https://doi.org/10.1126/science.1140746>.
- Ye, C., E. Pendall, and P. Ciais, 2015: Focus on extreme events and the carbon cycle. *Environ. Res. Lett.* **10** (7), <https://doi.org/10.1088/1748-9326/10/7/070201>.
- Zelinka, M.D., and Coauthors, 2020. Causes of higher climate sensitivity in CMIP6 models. *Geophys. Res. Lett.* **47**, e2019GL085782 & supp. matter, <https://doi.org/10.1029/2019GL0878>.